Inductosyn

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Introduction

This report, in the beginning, shows the Inductosyn operating principle and then in the second part describes the different solutions to obtain the estimations of actual angle and speed of the Inductosyn rotor.

Inductosyn position transducers

Operating principles

An Inductosyn transducer consists of two noncontacting elements, a scale and slider for the linear transducer and a rotor and stator for rotary transducer. Inductosyn position transducers are “printed circuit” form of electrical resolver. The printed circuit transducer patterns can be produced on almost any substrate material. The most common Inductosyn transducer application uses inductive coupling between the moving patterns. Through the following examples we propose to show the principle.

The induced voltage, $V_2$ is at the maximum amplitude when the winding conductors $W_2$ of one element are exactly aligned with the winding conductors $W_1$ of the other element.

The induced voltage, $V_2$ passes through zero when the winding conductors $W_2$, of one element are midway between the winding conductors, $W_1$, of the other element. The distance moved by $W_2$, from the first figure, is $\frac{x_p}{4}$.

The induced voltage, $V_2$ reaches a maximum amplitude of opposite phase when the winding conductors of both elements are in their next exactly aligned position. The distance moved by $W_2$, from the first figure, is $\frac{x_p}{2}$. 
The induced voltages, for intermediate positions to those shown before, have values defined by a sinusoidal function.

In actual practice, a second winding pattern is incorporated on one element such that it is displaced $\Delta x_p/4$ from the first winding pattern (shown below).

An AC excitation signal applied to the one-winding element results in two output signals from the two-winding element.
Rotary inductosyn transducers

Rotary Inductosyn rotor and stator elements are normally supplied as separate units for direct attachment to the equipments shaft and support. These sensors permit to reach very high accuracy.

The reference winding is fixed on the rotor, and therefore, it rotates jointly with the shaft passing the output windings as shown on the figure bellow.

Two stator windings are shifted of $\pi/4$ and generate the sine and co-sine voltages $U_{\text{sin}}, U_{\text{cos}}$, respectively.

The number of poles $p$ is, for the standard versions, fixed between 2 and 2048.

We obtain the following relations:

$$u_{\text{sin}} = \bar{U}_{\text{sin}} \cdot \sin(\omega_{\text{rot}} \cdot t) \cdot \sin \left( \frac{2\pi \cdot \text{pole}}{p} \right) = \bar{U}_{\text{sin}} \cdot \sin(\omega_{\text{rot}} \cdot t) \cdot \sin(\varphi_{\text{ref}})$$

$$u_{\text{cos}} = \bar{U}_{\text{cos}} \cdot \sin(\omega_{\text{rot}} \cdot t) \cdot \cos \left( \frac{2\pi \cdot \text{pole}}{p} \right) = \bar{U}_{\text{cos}} \cdot \sin(\omega_{\text{rot}} \cdot t) \cdot \cos(\varphi_{\text{ref}})$$
The reference winding is supplied by a sine signal.

Specified at a carrier frequency of 10 kHz, Inductosyn units have been operated at frequencies from 200 Hz to 200 kHz.

Recommended operating range for most units is 2.5 kHz to 100 kHz.

The frequency of the generated voltages is identical to the reference voltage and their amplitudes vary according to the sine and co-sine of the shaft angle $\Theta$.

Considering that one of the output windings is aligned with the reference winding, then it is generated full voltage on that output winding and zero voltage on the other output winding and vice versa.

The rotor angle $\Theta$ can be extracted from these voltages using a digital approach as will be discussed in the 2nd part of the report.
Incremental rotary Inductosyn transducers

Inductosyn transducers such that describe above must be considered as incremental sensors. In fact the angular position is a part of the mechanical position. They are so used for absolute position measure with a very little sector, or in conjunction with an absolute position mark (micro-switch for example).

Absolute rotary Inductosyn transducers

When the absolute angular position is necessary, we use Inductosyn sensor which have two distinct measure strips, one having a supplementary pole than other. In these cases, we have the following relations.

**Strip 1:** number of pole $p$

\[
\begin{align*}
\bar{u}_{\text{sin1}} &= \bar{U}_{\text{sin}} \cdot \sin(\omega_p t) \cdot \sin(p \cdot \vartheta_m) \\
\bar{u}_{\cos1} &= \bar{U}_{\cos} \cdot \sin(\omega_p t) \cdot \cos(p \cdot \vartheta_m)
\end{align*}
\]

**Strip 2:** number of pole $p+1$

\[
\begin{align*}
\bar{u}_{\text{sin2}} &= \bar{U}_{\text{sin}} \cdot \sin(\omega_{p+1} t) \cdot \sin((p+1) \cdot \vartheta_m) \\
\bar{u}_{\cos2} &= \bar{U}_{\cos} \cdot \sin(\omega_{p+1} t) \cdot \cos((p+1) \cdot \vartheta_m)
\end{align*}
\]

Thank to two distinct convertor, we can extract the electrical angular position of each strip then the absolute angular position.

The outputs of these two patterns are digitized and subtracted.

\[
\begin{align*}
\vartheta_{m} &= \vartheta_{m1} - \vartheta_{m2} \\
\vartheta_{m1} &= p \vartheta_m \\
\vartheta_{m2} &= (p+1) \vartheta_m
\end{align*}
\]

The result of this subtraction is digital coarse data which is then combined with one of the digitized fine outputs to provide an output that is again accurate and absolute.
Accuracy

Standard units have accuracies to:

\[ \pm 1 \text{ arc second} \]

Select units better than

\[ \pm 0.5 \text{ arc second} \]

Repeatability, for example in the case of Inductosyn 3.7 inch OD x 0.61 ID, is less than \[ \pm 0.2 \text{ arc second} \].
Basics of angle extraction

Modern systems use the digital approach to extract rotor angle and speed from the resolver output signals. The most common solution is either a Trigonometric or Angle Tracking Observer method. We propose here to see an overview of them.

Trigonometric

The shaft angle can be determined by an Inverse Tangent function of the quotient of the sampled resolver output voltages $U_{\sin}$, $U_{\cos}$.

This determination can be expressed, in terms of resolver output voltages, as follows:

$$\theta = \text{atan} \left( \frac{U_{\sin}}{U_{\cos}} \right)$$

An indispensable precondition of the accurate rotor angle estimation is to sample the resolver output signals simultaneously and close to their period peaks.

Note that our application requires knowledge of the rotor angle and the rotor speed to control the motor speed. The Trigonometric method, however, only yields values of the unfiltered rotor angle without any speed information. Therefore, for a final application, it is often required that a speed calculation with smoothing capability be added. This drawback might readily be eliminated if a special Angle Tracking Observer is utilized. This method is discussed in the next section.
Angle tracking observer

The second method (algorithm), widely used for estimation of the rotor angle and speed, is generally known as an Angle Tracking Observer.

![Block Scheme of the Angle Tracking Observer](image)

A great advantage of the Angle Tracking Observer method, compared to the Trigonometric method, is that it yields smooth and accurate estimations of both the rotor angle and rotor speed.

The Angle Tracking Observer compares values of the resolver output signals $U_{sin}$, $U_{cos}$ with their corresponding estimations $\hat{U}_{sin}$, $\hat{U}_{cos}$. As in any common closed-loop systems, the intent is to minimize observer error. The observer error is given here by subtraction of the estimated resolver rotor angle $\hat{\Theta}$ from the actual rotor angle $\Theta$.

Note that mathematical expression of observer error is known as the formula of the difference of two angles:

$$\sin (\Theta - \hat{\Theta}) = \sin(\Theta) \cdot \cos(\hat{\Theta}) - \cos(\Theta) \cdot \sin(\hat{\Theta})$$
Where \( \sin(\theta - \hat{\theta}) \) denotes observer error, \( \theta \) is the actual rotor angle and \( \hat{\theta} \) is its corresponding estimation.

In the case of small deviations of the estimated rotor angles compared to the actual rotor angle, the observer error may be considered in the form \( \theta - \hat{\theta} \).

The main benefit of the Angle Tracking Observer utilization, in comparison with the Trigonometric method, is its smoothing capability. Smoothing is achieved by the integrator and PI controller, which are connected in series and closed by a unit feedback loop, see the block diagram below. This block diagram nicely tracks actual rotor angle and speed and continuously updates their estimations.

The Angle Tracking Observer transfer function is expressed, with the help of its simplified block scheme, as follows:

\[
\frac{\hat{\theta}(s)}{\theta(s)} = \frac{K_1(1 + K_2s)}{s^2 + K_1K_2s + K_1}
\]

The characteristic polynomial of the Angle Tracking Observer corresponds to the denominator of transfer function:

\[
s^2 + K_1K_2s + K_1
\]

Appropriate dynamic behavior of the Angle Tracking Observer may be achieved by placement of the poles of the characteristic polynomial. This general method is based on matching the coefficients of the characteristic polynomial with the coefficients of the general second-order system \( G(s) \):

\[
G(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}
\]

Where \( \omega_n \) is the Natural Frequency and \( \zeta \) is the Damping Factor. Once the desired response of the general second-order system \( G(s) \) is found, the Angle Tracking Observer coefficients \( K_1, K_2 \) can be calculated using these expressions:
The Angle Tracking Observer transfer function is a second order and has one zero. This real-axis zero affects the residue, or amplitude, of a response component but does not affect the nature of the response - exponential damped sine. It can be proven that the closer the zero is to the dominant poles, the greater its effect is on the transient response. As the zero moves away from the dominant poles the transient response approaches that of the two-pole system.

Example

To better understanding, we propose to take an example:

Let take 3.7 inch size one which has got 360 poles.

At 1600 rpm (27 rps) we have a mechanical frequency to about 10 kHz (27 rps*360 poles).

We take a frequency excitation to 100 kHz. For this configuration the relation becomes:

\[ \omega_{\text{rot}} = \Omega_{\text{rot}} \sin(\omega_{\text{rot}} t) = \Omega_{\text{rot}} \sin(2 \times 100,000 \times 360,000 t) \cdot \sin(360, \omega_{\text{rot}} t) \]

\[ \omega_{\text{acc}} = \Omega_{\text{acc}} \sin(\omega_{\text{acc}} t) \cdot \cos\left(\frac{\omega_{\text{rot}} t}{\omega_{\text{rot}}}\right) = \Omega_{\text{acc}} \sin(2 \times 100,000 \times 360,000 t) \cdot \cos(360, \omega_{\text{rot}} t) \]

We have the following signal on out of sensor:
As you can see, if we do a measure every signal period, we will be able to do a measure only each 200 µm. To have more points, we will need to take several points on one signal period or increase the excitation signal which is not recommended.

Moreover, currently, for the commercial electronics, the acquisition frequency is about 10 kHz which is not enough, to increase this value; the real challenge will be in developing digitizing electronics.

When these problems will be resolved, we will know the position with a precision of about +/- 4 arc sec.

**Conclusion**

INDUCTOSYN have a number of existing units in the general size range which should have sufficient accuracy and/or repeatability to satisfy our requirement.

The capabilities for speed and acceleration are primarily dependent upon the characteristics of the digitizing electronics and not the transducer itself.

The real challenge will be in developing digitizing electronics with the correct combination of tracking speed and resolution since no currently available commercial electronics has the required combination of capabilities.

For the moment, two companies propose to work with us to develop a new digitizing electronic card, Inductosyn and Data Device Corporation (DDC).

The next step should be, to define a specification of our requirement in terms of operate frequencies, of resolution, of accuracy and of repeatability, and then, to contact the company.